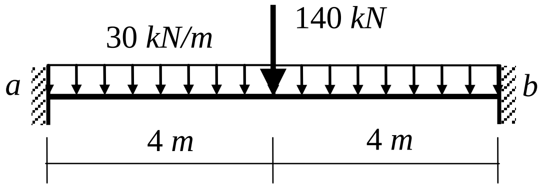
Answer of Second Semester Final Examination

- Attempt all questions.
- The Exam consists of **4** questions in **1** page.
- Maximum grade is **60 Marks**.

Question (1): (15 Marks)

Using the three-moments equation, draw the **shear force** and **bending moment** diagrams for the shown loaded beam.



- The simply supported moment diagram (M_O) on span ab is as shown.
- The moment at a_o, $M_{ao} = 0$ and the moment at b_o, $M_{bo} = 0$
- Applying three-moments equation for spans a_oa and ab:

$$M_{a_o}\left(\frac{0}{\infty}\right) + 2M_a\left(\frac{0}{\infty} + \frac{8}{I}\right) + M_b\left(\frac{8}{I}\right) = -6\left(\frac{0}{\infty} + \frac{640+560}{I}\right)$$

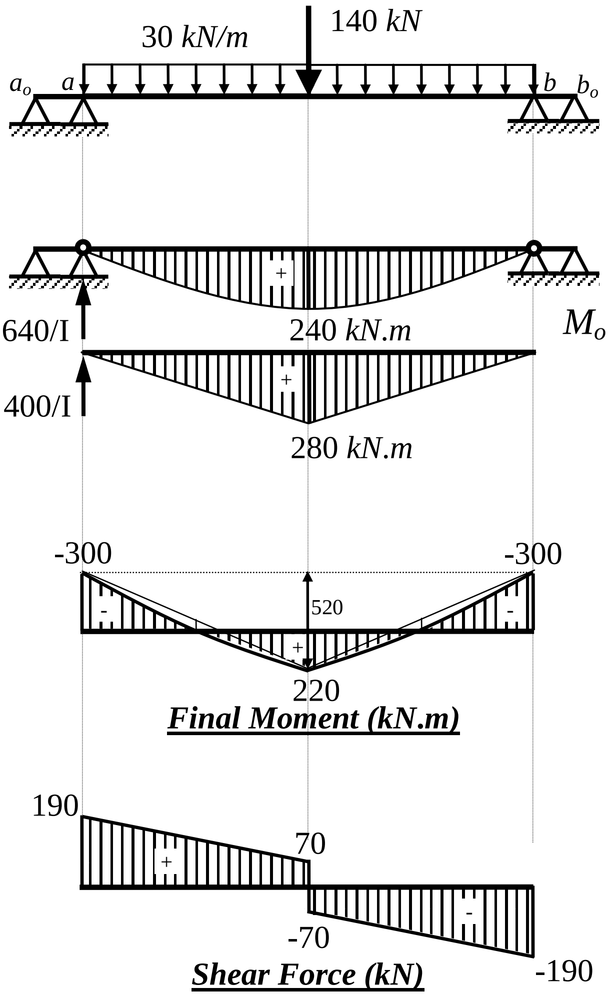
$$16M_a + 8M_b = -6\,(1200)$$

But from symmetry, $M_b = M_a$

$$16M_a + 8M_a = -6\,(1200)$$

$$\rightarrow 24M_a = -7200 \rightarrow \boxed{M_a = -300\ kN.m}$$

The bending moment and shear force diagram are as shown.



Final Moment (kN.m)

Shear Force (kN)

With my best wishes
Dr. M. Abdel-Kader

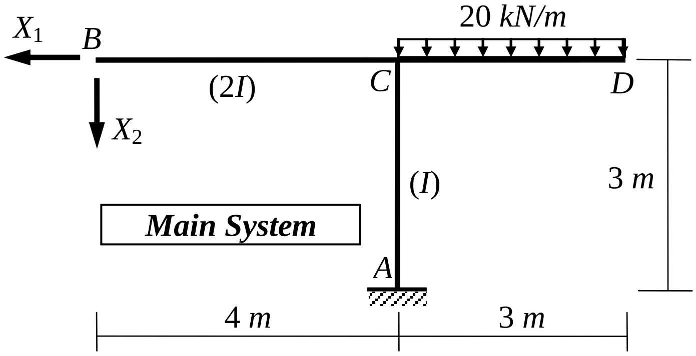
Question (2): (15 Marks)

For the shown loaded frame with variable moment of inertia, **using the virtual work method**,

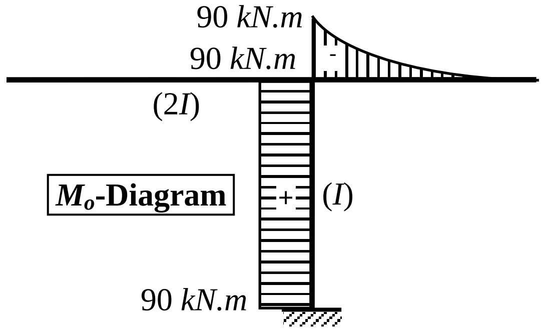
(a) find the reactions at the supports *A* and *B*.

(b) draw the bending moment diagram.

The relative moments of inertia are given between brackets and *E* is constant.



- Main system is as shown.
- Draw M_0-, M_1- and M_2-Diagram.
- Calculations of displacements.

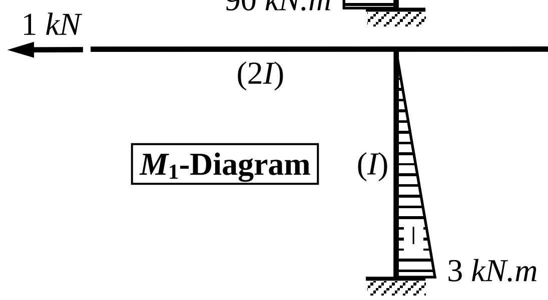


$$\delta_{10} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[(3\times 90)\left(-\tfrac{1}{2}\times 3\right)\right] = \frac{-405}{EI}$$

$$\boxed{\delta_{10} = \frac{-405}{EI}}$$

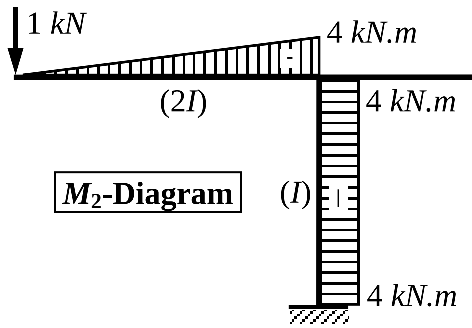
$$\delta_{20} = \int \frac{M_o M_2}{EI} dL = \frac{1}{EI}\left[(3\times 90)(-4)\right] = -\frac{1080}{EI}$$

$$\boxed{\delta_{20} = \frac{-1080}{EI}}$$



$$\delta_{12} = \delta_{21} = \int \frac{M_1 M_2}{EI} dL = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times 3\times 3\right)(-4)\right] = \frac{18}{EI}$$

$$\boxed{\delta_{12} = \delta_{21} = \frac{18}{EI}}$$



$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times 3\times 3\right)\left(-\tfrac{2}{3}\times 3\right)\right] = \frac{9}{EI}$$

$$\boxed{\delta_{11} = \frac{9}{EI}}$$

$$\delta_{22} = \int \frac{M_2 M_2}{EI} dL$$

$$= \frac{1}{2EI}\left[\left(-\tfrac{1}{2}\times 4\times 4\right)\left(-\tfrac{2}{3}\times 4\right)\right] + \frac{1}{EI}\left[(-3\times 4)(-4)\right] = \frac{58.667}{EI} \qquad \boxed{\delta_{22} = \frac{58.667}{EI}}$$

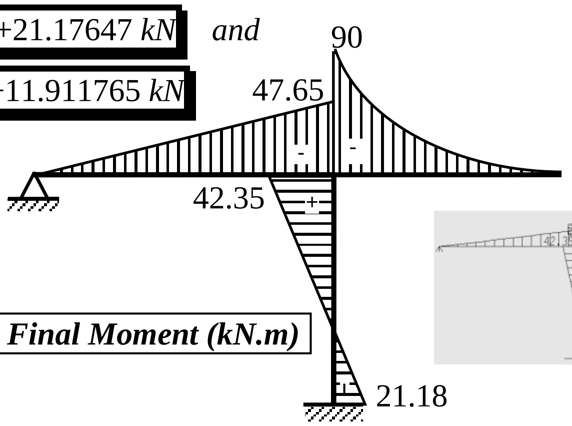
$$\delta_{10} + X_1\delta_{11} + + X_2\delta_{12} = 0$$

$$\delta_{20} + X_1\delta_{21} + + X_2\delta_{22} = 0$$

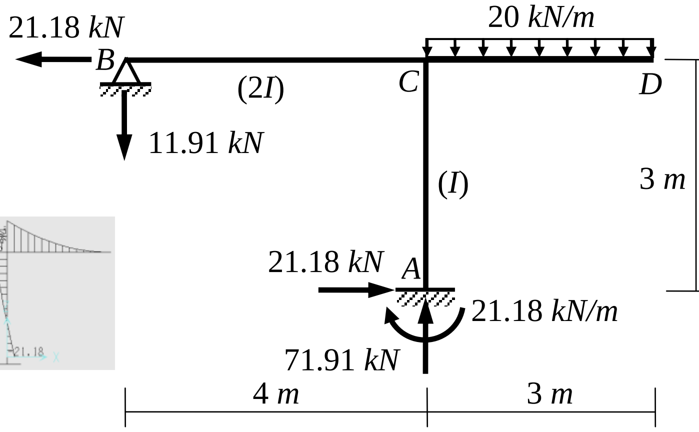
$$-405 + 9X_1 + 18X_2 = 0 \quad \rightarrow \times\text{-2} \quad 810 - 18X_1 - 36X_2 = 0$$

$$-1080 + 18X_1 + 58.667X_2 = 0 \qquad -1080 + 18X_1 + 58.667X_2 = 0$$

$\therefore$ $\boxed{X_1 = +21.17647\ kN}$ *and* $\boxed{X_2 = +11.911765\ kN}$



Final Moment (kN.m)



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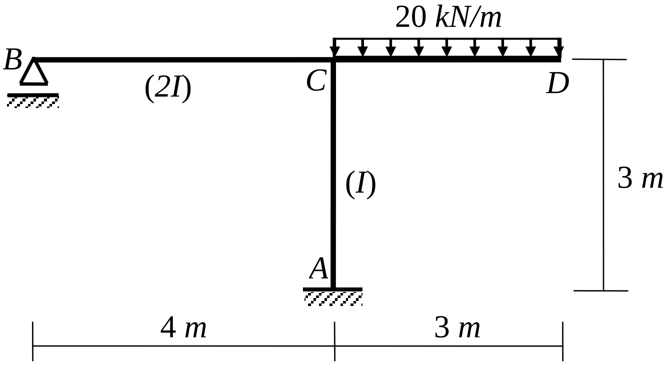
Question (3): (15 Marks)



For the shown loaded frame with variable moment of inertia, **using the slope deflection method**,

(a) find the rotation at C (θ_C) and the sway of the frame Δ.

(b) draw the bending moment diagram.

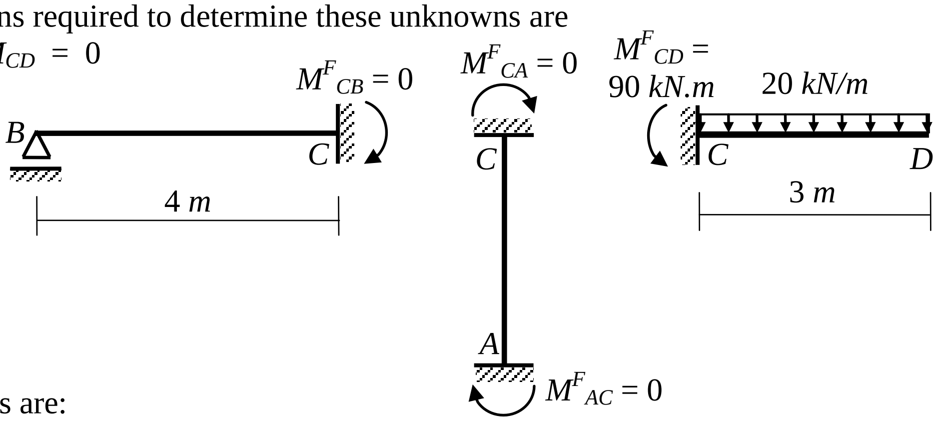
The relative moments of inertia are given between brackets and E is constant.

1- Unknown displacements: θ_C and Δ.

2- The static equilibrium equations required to determine these unknowns are

$$\sum M_C = M_{CA} + M_{CB} + M_{CD} = 0$$
$$\sum F_x = 0$$

3- Fixed end moments:



4- The slope deflection equations are:

$$M_{CA} = M^F_{CA} + \frac{2EI}{L}\left(2\theta_C + \theta_A + 3\psi_{CA}\right) = 0 + \frac{2E(I)}{3}\left(2\theta_C + 0 - \frac{3\Delta}{3}\right) = \tfrac{4}{3}EI\theta_C - \tfrac{2}{3}EI\Delta$$

$$M_{CB} = M^F_{CB} + \frac{3EI}{L}\left(\theta_C + \psi_{CB}\right) = 0 + \frac{3E(2I)}{4}\left(\theta_C + 0\right) = \tfrac{3}{2}EI\,\theta_C$$

$$M_{CD} = -90\ kN.m$$

$$M_{AC} = M^F_{CA} + \frac{2EI}{L}\left(2\theta_A + \theta_C + 3\psi_{CA}\right) = 0 + \frac{2E(I)}{3}\left(0 + \theta_C - \frac{3\Delta}{3}\right) = \tfrac{2}{3}EI\theta_C - \tfrac{2}{3}EI\Delta$$

5- Substituting into the static equilibrium equations,

$$\sum M_C = M_{CA} + M_{CB} + M_{CD} = \tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta - 90 = 0$$

$$\boxed{\tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta = 90 \quad \ldots..(1)}$$

$$\sum F_x = X_A = 0$$

$$X_A = (M_{AC} + M_{CA})/L_{AC} = \left(\tfrac{4}{3}EI\theta_C - \tfrac{2}{3}EI\Delta + \tfrac{2}{3}EI\theta_C - \tfrac{2}{3}EI\Delta\right)/3 = 0$$

$$\boxed{\tfrac{2}{3}EI\theta_C - \tfrac{4}{9}EI\Delta = 0 \quad \ldots..(2)}$$

6-
$$\tfrac{17}{6}EI\theta_C - \tfrac{2}{3}EI\Delta = 90 \quad \ldots..(1)$$

Eq.(2) $\times \frac{3}{2} \rightarrow$
$$EI\theta_C - \tfrac{2}{3}EI\Delta = 0 \quad \ldots..(2')$$

Then $\boxed{\theta_C = \frac{540}{11EI} = \frac{49.09}{EI}}$ and $\boxed{\Delta = \frac{810}{11EI} = \frac{73.64}{EI}}$

7- Back-substituting by θ_C and Δ into the slope deflection equations, the end moments become:

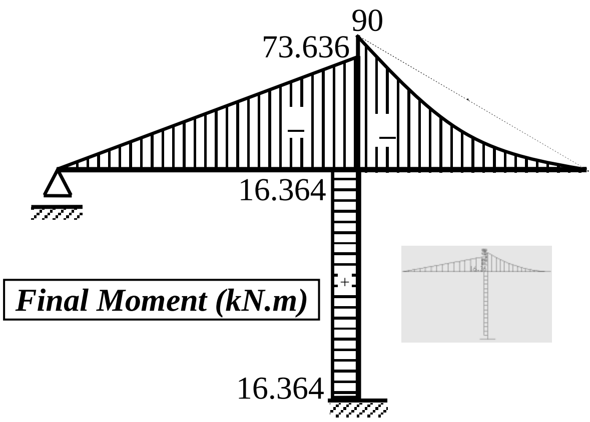
$M_{CA} = \frac{4}{3}EI\theta_C - \frac{2}{3}EI\Delta$ = 16.364 $kN.m$

$M_{CB} = \frac{3}{2}EI\,\theta_C$ = 73.636 $kN.m$

M_{CD} = -90 $kN.m$

$M_{AC} = \frac{2}{3}EI\theta_C - \frac{2}{3}EI\Delta$ = -16.364 $kN.m$

8- The final bending moment diagram for the whole frame is as shown.



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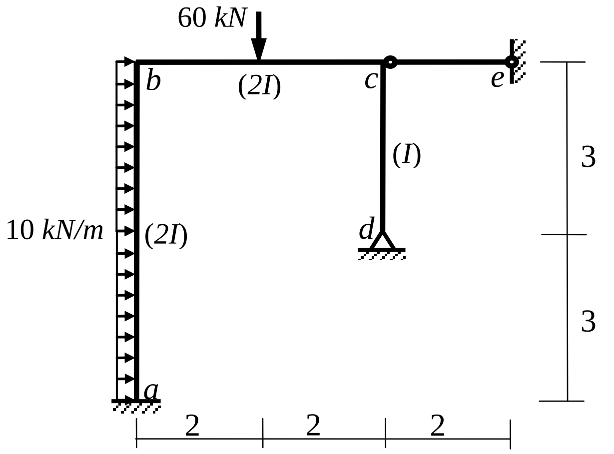
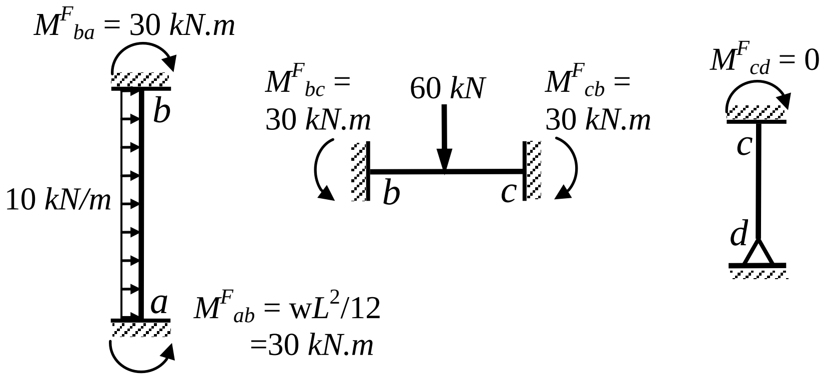
Question (4): (15 Marks)

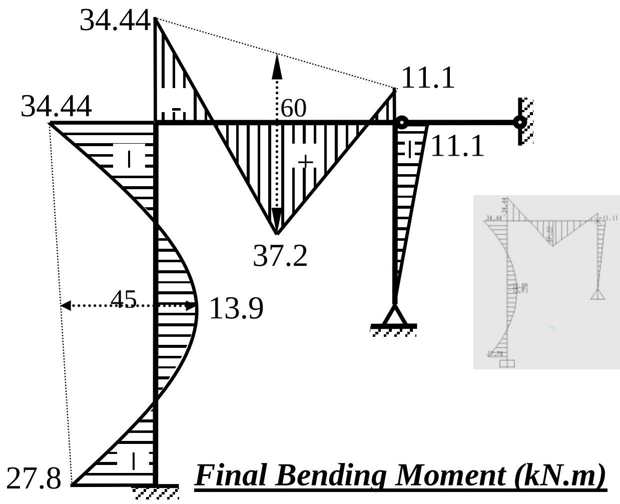
Using the moment distribution method, draw the bending moment diagram for the shown loaded frame with variable moment of inertia.

E is constant. The relative moments of inertia are given between brackets..



- Fixed end moments:





Final Bending Moment (kN.m)

- Distribution factors (D.F.)

At Joint b

$$k_{ba} = \frac{4EI}{L_{ba}} = \frac{8EI}{6} = \frac{4}{3}EI \quad \text{and} \quad k_{bc} = \frac{4EI}{L_{bc}} = \frac{4E(2I)}{4} = 2EI \qquad \sum k_i = k_{ba} + k_{bc} = \frac{10}{3}EI$$

$$D.F._{ba} = \frac{k_{ba}}{\sum k_i} = 0.4 \quad \& \quad D.F._{bc} = \frac{k_{bc}}{\sum k_i} = 0.6$$

$$\boxed{D.F._{ba} = 0.4 \qquad D.F._{bc} = 0.6}$$

At Joint *c*

$$k_{cb} = \frac{4EI}{L_{cb}} = \frac{4E(2I)}{4} = 2EI \quad \text{and} \quad k_{cd} = \frac{3EI}{L_{cd}} = \frac{3E(I)}{3} = EI \qquad \sum k_i = k_{cb} + k_{cd} = 3EI$$

$$D.F._{cb} = \frac{k_{cb}}{\sum k_i} = \frac{2}{3} \quad \& \quad D.F._{cd} = \frac{k_{cd}}{\sum k_i} = \frac{1}{3}$$

$$\boxed{D.F._{cb} = 2/3 \qquad D.F._{cd} = 1/3}$$

Joint	*a*	*b*		*c*		*d*
Member	*ab*	*ba*	*bc*	*cb*	*cd*	*dc*
D.F.	1	0.4	0.6	2/3	1/3	0
F.E.M.	-30	+30	-30	+30	0	0
B.M.				-20	-10	
C.O.M.			-10			0
B.M.		+4	+6			
C.O.M.	+2			+3		
B.M.				-2	-1	
C.O.M.			-1			0
B.M.		+0.4	+0.6			
C.O.M.	+0.2			+0.3		
B.M.				-0.2	-0.1	
C.O.M.			-0.1			0
B.M.		+0.04	+0.06			
C.O.M.	+0.02			+0.03		
B.M.				-0.02	-0.01	
Final M	-27.78	+34.44	-34.44	+11.11	-11.11	0

With my best wishes

Dr. M. Abdel-Kader